Geometry and design of involute spur gears with asymmetric teeth

Alexander Kapelevich

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Abstract

This paper presents a method of research and design of gears with asymmetric teeth that enables to increase load capacity, reduce weight, size and vibration level. The method is illustrated with numerical examples.

1. Introduction

Conventional involute spur gears are designed with symmetric tooth side surfaces [l, 2]. It is well known that the conditions of load and meshing are different for drive and coast tooth's side. Application of asymmetric tooth side surfaces enables to increase the load capacity and durability for the drive tooth side. Therefore, the geometry and design of asymmetric spur gears represents an important problem.

There are several articles about involute gears with asymmetric, or so-called, buttress teeth [3, 4]. They consider the low pressure angle profile (as a rule 20°) for the drive side and high pressure angle profile for the coast side teeth. Such an approach enables to decrease the bending stresses and keeps contact stresses on the same level as for symmetric teeth with equal pressure angle. However, this design is accompanied by raising mesh stiffness, increasing noise and vibration with frequency of the cycle of meshing. It does not affect the load capacity limited by contact stresses.

Nomenclature

a	generating rack tooth addendum, mm (in.)
b	generating rack tooth dedendum, mm (in.)
C	center distance, mm (in.)
c	radial clearance, mm (in.)
C_S	tooth thickness coefficient
D_b	base circle diameter, mm (in.)
D_Δ	tip circle diameter, mm (in.)
D_o	outside circle diameter, mm (in.)
D_p	pitch diameter, mm (in.)
D_{pr}	generating pitch diameter, mm (in.)
D_r	root diameter, mm (in.)
k	coefficient of asymmetry
m_G	gear ratio
m_o	top land thickness coefficient
m_p	contact ratio
p	diametral pitch, 1/mm (1/in.)
p_b	diametral pitch on the base circle, 1/mm (1/in.)
p_r	generating rack pitch, 1/mm (1/in.)
R	generating rack tooth radius, mm (in.)
S_o	top land thickness, mm (in.)
S_p	tooth thickness on pitch diameter, mm (in.)
S_{pr}	generating rack tooth thickness, mm (in.)
ϕ	pressure angle, deg
ϕ_f	profile angle in the bottom contact point, deg
ϕ_o	profile angle on outside circle
ϕ_r	generating rack profile angle, deg.
ν	profile angle in the intersection (tip) point of the two involutes, deg

Subscript

c	coast involute profile
d	drive involute profile
1	pinion (sun gear)
2	gear (planet gear)
3	ring gear

The approach developed in this paper differs from the previous ones [3, 4] and is based on the following considerations:

1. For an external gearing a larger pressure angle is proposed for the drive tooth side but not for the coast tooth side.

2. The parameters of asymmetric gears are defined independently from any generating rack parameters. On the contrary, the choice of the generating rack (or racks) parameters is based on results of the asymmetric gear synthesis.

The load capacity of the involute gear greatly depends on the pressure angle. The growth of the drive side pressure angle increases radii of curvature, reducing Hertzian contact stress, and enlarges the thickness of the central elastohydrodynamic film. It is in agreement with the tendency of design of aerospace gears with a larger pressure angle (25°-28° instead of 20° [5]).

The analysis of conventional involute spur gears is based on parameters of the standardized generating rack and its shift. The use of this method leaves a lot of gear combinations out of consideration because an area of the selection is limited by standard parameters of the generating rack. In another approach, which is developed by Prof. E.B. Vulgakov in his theory of `generalized parameters', the gears are described independently from any generating rack [6]. The author has adopted this theory for gears with asymmetric teeth that are nonstandard [7-9] and developed the procedure of design and computation of gear geometry that is supported by the computer programs.

The content of the paper is organized as follows: Section 2 gives an analysis of asymmetric gearing. In Section 3, determination of the area of design parameters from the condition of existence of asymmetric gearing is given. Section 4 describes the synthesis of asymmetric spur gears with subsequent determination of generating rack parameters and Section 5 gives the numerical examples, prototypes, application.

The definitions and symbols used in this paper (see the Nomenclature) have been taken mainly from standard AGMA 1012-F90. Subscript symbols d (drive) and c (coast) regard to the sides with high and low pressure angle.

2. Analysis of asymmetric gearing

An asymmetric involute tooth is formed by two involutes of different base circles D_{bd} and D_{bc}, outside circle diameter D_o and fillet, which appears when generating profile is selected (see Fig. 1).

The profile angles in the intersection point of the two involutes (tip angles) are

$$\nu_d = \arccos\left(\frac{D_{bd}}{D_\Delta}\right), \qquad \nu_c = \arccos\left(\frac{D_{bd}}{D_\Delta}\right), \tag{1}$$

where Do is tip circle diameter. The coefficient of asymmetry is

$$k = \frac{D_{bc}}{D_{bd}} = \frac{\cos \nu_c}{\cos \nu_d} = \frac{\cos \phi_{bc}}{\cos \phi_{bd}}. \tag{2}$$

The symmetric teeth have k = 1.0. The profile angles on the outside diameter D_o are

$$\phi_{od} = \arccos\left(\frac{D_{bd}}{D_o}\right), \qquad \phi_{oc} = \arccos\left(\frac{D_{bc}}{D_o}\right). \tag{3}$$

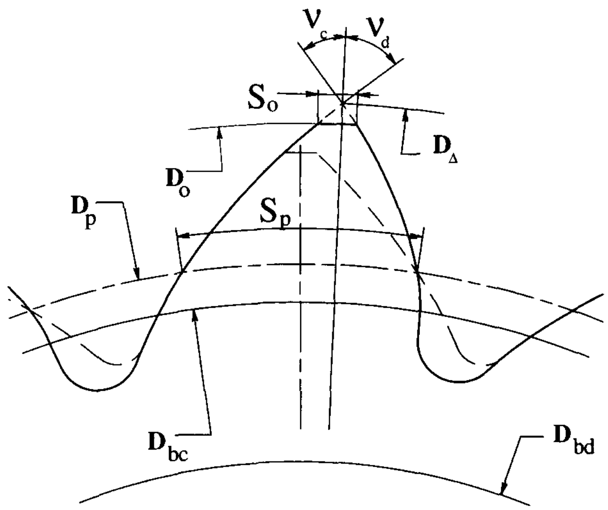

Fig. 1. Formation of the asymmetric involute tooth.

The top land thickness coefficient is

$$m_o = \frac{S_o}{D_{bd}} = \frac{\text{inv } \nu_d + \text{inv } \nu_c - \text{inv } \phi_{od} - \text{inv } \phi_{oc}}{2\cos \phi_{od}}, \qquad (4)$$

where inv x = tan x - x is involute function of the angle x, S_o is the top land thickness. The coefficient m_o is selected within (0.25-0.4)/N range, where N is number of teeth. The coefficient m_o should be large enough to avoid fracturing of the tip for the case-hardened gears, but the increase of m_o reduces the transverse contact ratio. Fig. 2 shows a zone of the tooth action for the asymmetric gears. The pressure angles can be fund from

$$\text{inv } \phi_d + \text{inv } \phi_c = \frac{\text{inv } \nu_{1d} + \text{inv } \nu_{1c} + m_G (\text{inv } \nu_{2d} + \text{inv } \nu_{2c}) - 2\pi/N_1}{1 + m_G} \qquad (5)$$

and Eq. (2). The contact ratios are: for drive sides:

$$m_{pd} = \frac{N_1\left(\tan \phi_{o1d} + m_G \tan \phi_{o2d} - (1 + m_G)\tan \phi_d\right)}{2\pi},$$

for coast sides:

$$m_{\mathrm{pc}} = \frac{N_1\left(\tan\phi_{\mathrm{o1c}} + m_{\mathrm{G}}\tan\phi_{\mathrm{o2c}} - (1+m_{\mathrm{G}})\tan\phi_{\mathrm{c}}\right)}{2\pi}, \tag{7}$$

To avoid the interference, the profile angles in the bottom points of the coast sides (they are more sensitive) must be larger or equal to zero: for the pinion:

$$\tan\phi_{\mathrm{f1c}} = (1+m_{\mathrm{G}})\tan\phi_{\mathrm{c}} - m_{\mathrm{G}}\tan\phi_{\mathrm{o2c}} \ > \ 0, \tag{8}$$

• for the gear:

$$\tan\phi_{\mathrm{f2c}} = \frac{(1+m_{\mathrm{G}})\tan\phi_{\mathrm{c}}}{m_{\mathrm{G}}} - \frac{\tan\phi_{\mathrm{o1c}}}{m_{\mathrm{G}}} \ > \ 0. \tag{9}$$

The profile angles in the bottom points of the drive sides are

$$\tan\phi_{\mathrm{f1d}} = (1+m_{\mathrm{G}})\tan\phi_{\mathrm{d}} - m_{\mathrm{G}}\tan\phi_{\mathrm{o2d}},$$

$$\tan\phi_{\mathrm{f2d}} = (1+m_{\mathrm{G}})\frac{\tan\phi_{\mathrm{d}}}{m_{\mathrm{G}}} - \frac{\tan\phi_{\mathrm{o1d}}}{m_{\mathrm{G}}}. \tag{10}$$

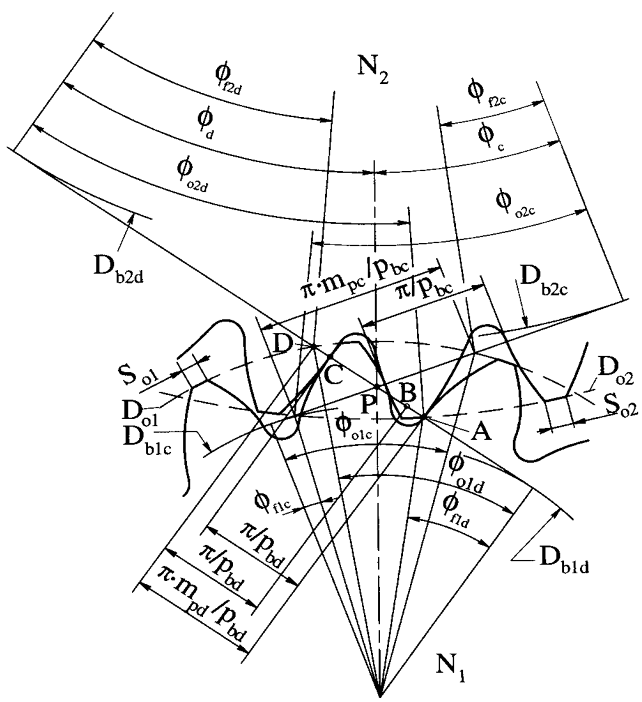

Fig. 2. The zone of the tooth action for the pinion and the gear with asymmetric teeth.

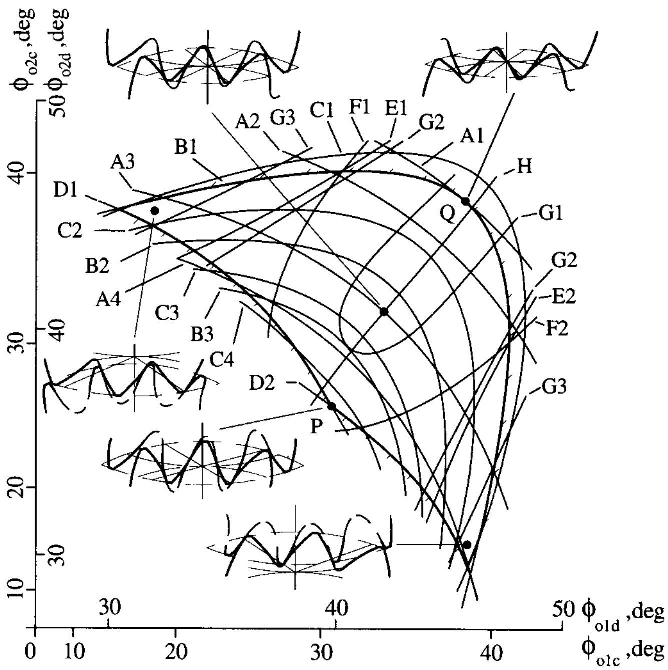

Fig. 3. The sample of the area of existence for couples of the pinion and the gear with asymmetric teeth.

3. Area of existence of asymmetric gears

An area of existence is the locus of possible design solutions for the pinion and the gear if number of teeth N_1,N_2 with the coefficient of asymmetry k and the top land thickness coefficients m_{o1},m_{o2} are given. This area is built in ϕ_{o1d}–ϕ_{o2d} coordinates and includes the number of isograms reflecting the constant values of the different gear parameters. The area of existence of spur asymmetrical gears is limited by isograms of the contact ratios for the drive sides $m_{pd} = 1.0$,$m_{pc} = 1.0$ and interference isograms $\phi_{f1c} = 0$,$\phi_{f2c} = 0$. Each point of the area of existence corresponds to the couple of the gears with some dimensionless properties. These properties are pressure angles, contact ratios, pitting resistance geometry factor I, specific sliding ratio (slide/roll), etc.

A sample of the area of existence for couples of asymmetric gears $N_1 = 23$,$N_2 = 28$,$k = 1.12$,$m_{o1} = 0.012$,$m_{o2} = 0.015$ is shown in Fig. 3. The isograms A show constant pressure angles: isogram A1 is $\phi_d = 42^\circ$ ($\phi_c = 33.7^\circ$), A2 is $\phi_d = 40^\circ$ ($\phi_c = 30.9^\circ$), A3 is $\phi_d = 36^\circ$ ($\phi_c = 25^\circ$), and A4 is $\phi_d = 33^\circ$ ($\phi_c = 20^\circ$). The isograms B and C show constant contact ratios for d- and c-sides correspondingly: isogram B1 is $m_{pd} = 1.0$, B2 is $m_{pd} = 1.2$, B3 is $m_{pd} = 1.4$, C1 is $m_{pc} = 1.0$, C2 is $m_{pc} = 1.2$, C3 is $m_{pc} = 1.6$, and C4 is $m_{pc} = 2.0$. The isograms D1 and D2 show the interference conditions $\phi_{f1c} = 0$ and $\phi_{f2c} = 0$. The isograms E1 ($\phi_{o1d} = \phi_d$), E2 ($\phi_o 2d = \phi_d$), and F1, F2 describe pitch point location with the reference to the line of action.

The gears that located between isograms E1 and E2 have pitch point on the line of action, the rest of the gears have pitch point is out of the line of action. The gears that located between isograms F1 and F2 have pitch point on the line of action where one-couple-of-teeth contact takes place. The gears that are between isograms F1 and E1 or F2 and E2 have pitch point is on the line of action where two-couple-of-teeth contact takes place. The isograms G show constant power losses if friction coefficient was taken $f = 0.05$: G1 is $P_t = 1.25\%$, G2 is $P_t = 2.5\%$, G1 is $P_t = 5\%$. The isogram H describes couples that have the same (balanced) specific sliding ratio in the top and bottom points of involute profile for the pinion and the gear. Point Q represents the couple with maximum pressure angle $\phi_{\mathrm{d}} = 41.9^\circ$ ($\phi_{\mathrm{c}} = 33.5^\circ$) and minimum contact ratio $m_{\mathrm{pd}} = 1.0$ ($m_{\mathrm{pc}} = 1.06$). Point P represents couple with minimum pressure angle $\phi_{\mathrm{d}} = 30.4^\circ$ ($\phi_{\mathrm{c}} = 15^\circ$) and maximum contact ratio $m_{\mathrm{pd}} = 1.60$ ($m_{\mathrm{pc}} = 2.16$). There are also shown different couples of the gears corresponding to different points of the area of existence.

4. Synthesis of asymmetric gears and generating rack parameters

The initial data for the synthesis of a couple of the asymmetric gears with numbers of the teeth N_1,N_2, coefficient of asymmetry k, top land thickness coefficient $m_{\mathrm{o}1}$,$m_{\mathrm{o}2}$ could be taken from the area of existence as a particular point. The coordinates of this point (the drive profile angles on the outside diameters ϕ_{o1d} and ϕ_{o2d}) and scale factor, such as diametral pitch p or center distance C, completely describe all geometry of the gearing. When coordinates ϕ_{o1d},ϕ_{o2d} are known, the coast profile angles on the outside diameters ϕ_{o1c},ϕ_{o2c} and the tip angles $v_{1\mathrm{d}}$,$v_{2\mathrm{d}}$,$v_{1\mathrm{c}}$,$v_{2\mathrm{c}}$ can be found from Eqs. (2) and (4), the pressure angles ϕ_{d},ϕ_{c}, from Eq. (5), the contact ratios m_{pd},m_{pc} from Eqs. (6) and (7), the profile angles in the bottom points ϕ_{f1c},ϕ_{f2c},ϕ_{f1d},ϕ_{f2d} from Eqs. (2), (8) and (10).

If the diametral pitch p is given then the center distance is

$$C = \frac{N_1 + N_2}{2p}. \tag{11}$$

If the center distance C is given then the diametral pitch is

$$p = \frac{N_1 + N_2}{2C}. \tag{12}$$

The pitch diameters of the pinion and the gear are

$$D_{\mathrm{p}1} = \frac{N_1}{p}, \qquad D_{\mathrm{p}2} = \frac{N_2}{p}. \tag{13}$$

The base diameters are

$$D_{\mathrm{b1d}} = D_{\mathrm{p}1}\cos\phi_{\mathrm{d}}, \qquad D_{\mathrm{b2d}} = D_{\mathrm{p}2}\cos\phi_{\mathrm{d}},$$

$$D_{\mathrm{b1c}} = D_{\mathrm{p}1}\cos\phi_{\mathrm{c}}, \qquad D_{\mathrm{b2c}} = D_{\mathrm{p}2}\cos\phi_{\mathrm{c}}. \tag{14}$$

Tooth thicknesses at the pitch diameters of the pinion and the gear are

$$S_{p1} = \frac{D_{p1}(\text{inv } \nu_{1d} + \text{inv } \nu_{1c} - \text{inv } \phi_d - \text{inv } \phi_c)}{2},$$

$$S_{p2} = \frac{D_{p2}(\text{inv } \nu_{2d} + \text{inv } \nu_{2c} - \text{inv } \phi_d - \text{inv } \phi_c)}{2}. \qquad (15)$$

If there is no area of existence for particular couple of the gears (N_1,N_2,k,m_{o1},m_{o2}) and angles ϕ_{o1d},ϕ_{o2d} are not known, the pressure angle ϕ_d or the contact ratio m_{pd} for drive sides could be selected as initial data. In this case, the tooth thickness coefficient $C_S = S_{p1}/S_{p2}$ must be chosen. For preliminary calculation C_S may be equal to square root of the gear ratio $m_G^{0.5}$. The tooth thicknesses S_{p1},S_{p2} will be defined more precisely during the generating rack (or racks) optimization.

Then tooth thicknesses at the pitch diameters of the pinion and the gear are:

$$S_{p1} = \frac{\pi C_S}{p(C_S + 1)}, \qquad S_{p2} = \frac{\pi}{p(C_S + 1)}. \qquad (16)$$

If the pressure angle ϕ_d is selected the tip angles $\nu_{1d},\nu_{2d},\nu_{1c},\nu_{2c}$ can be found from Eqs. (2) and (15), angles ϕ_{o1d},ϕ_{o2d} from Eq. (4). If contact ratio m_{pd} is selected the angles $\phi_d,\phi_c,\nu_{1d},\nu_{2d},\nu_{1c},\nu_{2c},\phi_{o1d},\phi_{o2d},\phi_{o1c},\phi_{o2c}$, can be found from joint solution of Eqs. (2), (4), (6) and (15).

The generating rack (see Fig. 4) must ensure formation of the active involute profiles without undercut and the acceptable radial clearance between the root circle and the outside

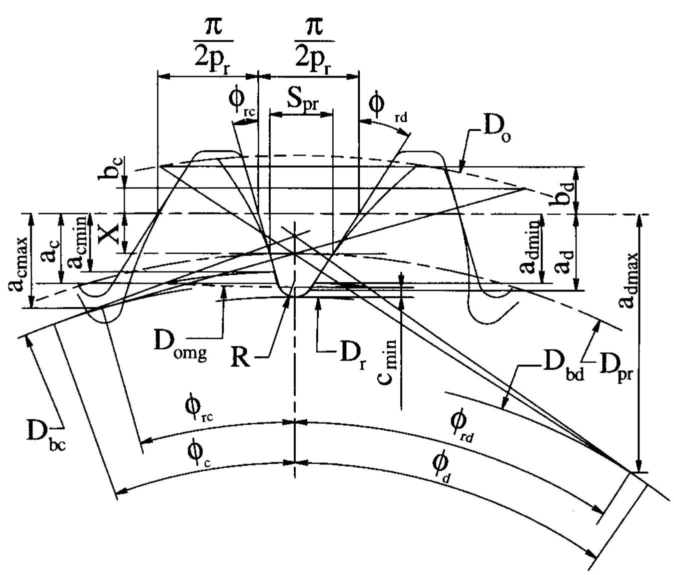

Fig. 4. The asymmetric generating rack.

circle of mating gear. In addition, the fillet shape should provide needed bending resistance and tooth stiffness. The generating rack base pitches have to be the same as on the gear:

$$p_{\rm bd} = \frac{p}{\cos \phi_{\rm d}} = \frac{p_{\rm r}}{\cos \phi_{\rm rd}} \quad \text{and} \quad p_{\rm bc} = \frac{p}{\cos \phi_{\rm c}} = \frac{p_{\rm r}}{\cos \phi_{\rm rc}}, \tag{17}$$

where $p_{\rm r}$ is the generating rack pitch, $\phi_{\rm rd}$ is generating rack angle for the drive side, $\phi_{\rm rc}$ is generating rack angle for the coast side.

The minimum drive side rack angle is $\phi_{\rm rd} = \arccos(1/k)$. In this case the coast side rack angle is $\phi_{\rm rc} = 0$. The increase of the rack angles reduces the rack round radius (or radii). The maximum value of the drive side rack angle is limited by the rack radius $R = 0$. For preliminary computation, the rack angle for the drive side is selected to be equal or less than the gearing profile angle $\phi_{\rm rd}$. It is changed during the generating rack optimization.

The rack angle for the coast side is

$$\phi_{\rm rd} = \arccos\left(k \cos \phi_{\rm bd}\right). \tag{18}$$

The generating pitch diameter is

$$D_{\rm pr} = \frac{N}{p_{\rm r}}. \tag{19}$$

The root diameter is

$$D_{\rm r} = 2\left(C - \frac{D_{\rm omg}}{2} - c_{\min}\right), \tag{20}$$

where $D_{\rm omg}$ is outside diameter of the mating gear, $c_{\min}$ is minimum radial clearance, $c_{\min} = (0.15\text{–}0.2)/p$.

The generating rack tooth thickness at the pitch line is equal to the arc space between teeth at the generating pitch circle

$$S_{\rm pr} = \frac{D_{\rm pr}\left(2\pi/N + \operatorname{inv} \phi_{\rm rd} + \operatorname{inv} \phi_{\rm rc} - \operatorname{inv} v_{\rm d} - \operatorname{inv} v_{\rm c}\right)}{2}. \tag{21}$$

The generating rack shift (the position to the generated gear) is

$$X = \frac{\pi/2p_{\rm r} - S_{\rm pr}}{\tan \phi_{\rm rd} + \tan \phi_{\rm rc}}. \tag{22}$$

The generating rack tooth rounding can be presented as the single arc, two arcs, two arcs with the straight line or some curve. The paper considers the single arc tooth rounding (see Fig. 4). The radius of the arc is

$$R = \frac{\left(\tan \phi_{\rm rd} + \tan \phi_{\rm rc}\right)\left(D_{\rm pr}/2 - D_{\rm r}/2 + X\right) - \pi/2p_{\rm r}}{\tan \phi_{\rm rd} + \tan \phi_{\rm rc} - 1/\cos \phi_{\rm rd} - 1/\cos \phi_{\rm rc}}. \tag{23}$$

The generating rack tooth addendums are

$$a_d = R\sin\phi_{rd} + \frac{\pi/2p_r - R\cos\phi_{rd}(1+k)/k}{\tan\phi_{rd} + \tan\phi_{rc}},$$

$$a_c = R\sin\phi_{rc} + \frac{\pi/2p_r - R\cos\phi_{rc}(1+k)/k}{\tan\phi_{rd} + \tan\phi_{rc}}. \tag{24}$$

In order to provide the involute profiles a_d and a_c must satisfy to complete profile forming conditions:

$$a_d > a_{dmin} = \frac{D_{bd}\sin\phi_{rd}(\tan\phi_{rd} - \tan\phi_{fd})}{2} + X,$$

$$a_c > a_{cmin} = \frac{D_{bc}\sin\phi_{rc}(\tan\phi_{rc} - \tan\phi_{fc})}{2} + X \tag{25}$$

and conditions to avoid the tooth undercut:

$$a_d < a_{dmax} = \frac{D_{bd}\sin\phi_{rd}\tan\phi_{rd}}{2} + X,$$

$$a_c < a_{cmax} = \frac{D_{bc}\sin\phi_{rc}\tan\phi_{rc}}{2} + X. \tag{26}$$

The generating rack tooth dedendums are

$$b_d = \frac{D_{bd}\sin\phi_{rd}(\tan\phi_{od} - \tan\phi_{rd})}{2} - X,$$

$$b_c = \frac{D_{bc}\sin\phi_{rc}(\tan\phi_{oc} - \tan\phi_{rc})}{2} - X. \tag{27}$$

The values of the profile angles ϕ_{rd},ϕ_{rc} and the shape of rack tooth rounding are subjects to optimization. The goal of the optimization is providing the minimum bending stress and needed tooth flexibility under a load. The 2D FEA was applied for the optimization in the thesis [7].

The generating rack parameters for the pinion and the gear are optimized independently and, as a rule, they are different. However, sometimes it is necessary to apply single generating rack for both the pinion and the gear. This compromising solution usually leads to higher bending stresses. The single generating rack parameters are optimized for the pinion and the gear simultaneously by selecting the proper values for the common rack profile angle ϕ_{rd} and the radial clearances c_1,c_2. It is recommended to compare bending stresses and tooth deflections for the pinion and the gear which are generated by a single rack and two individual racks to evaluated the possibility of single rack application.

5. Numerical examples, prototypes, application

Fig. 5 and Table 1 present design, results of stress analysis and vibro-testing of the single, stage generator gear with standard helical and asymmetric spur teeth. Two sets of asymmetric gears were made in order to test both high and low pressure angle drive sides for a comparison. The two accelerometers were attached to the gear housing in horizontal and vertical directions in the plane of the bearings. The average vibration level was registered. The standard and experimental gears were carburized, heat treated (the surface hardness $R_c > 55$), and the tooth surfaces were ground (AGMA Q11-Q12).

The two-stage planetary gear reducer of the turbo-prop engine TV7-117 (Design Bureau named after V.Y. Klimov, St. Petersburg, Russia) for Ilyushin-114 plane (currently in production) has all gears with asymmetric teeth. The photos and the data of that gear reducer are presented in Fig. 6 and Table 2, respectively.

In the planetary gear, the side of the planet gear teeth, that is a drive in the external mesh with the sun gear, becomes coast side in the internal mesh with the ring gear. It is preferred to have the high pressure angle contact in the external mesh and low pressure angle contact in the internal mesh because the internal gearing contact surface durability is significantly higher than in the external gearing.

The pressure angles in the internal mesh of planet and ring gears are:

- for drive sides:

$$\phi_{d_{2-3}} = \arccos\left(\frac{k \cos \phi_{d_{1-2}}(N_3 - N_2)}{(N_1 + N_2)}\right), \quad (28)$$

Fig. 5. The experimental generator gear

Table 1
The experimental generator gear data

Maximum transmitted power (hp)		77		
Pinion (RPM)		27000		
Center distance (mm)	C	74.25		
Gear set		Standard	Experimental 1	Experimental 2
Pinion tooth number	N_1	17	13	13
Gear tooth number	N_2	79	60	60
Gear width (mm)	W	22	22	22
Helix angle (deg)	β	14.5	0	0
Coefficient of asymmetry	k	1.0	1.26	0.79
Pressure angle for drive sides (deg)	ϕ_d	20.36	41	18
Pressure angle for coast sides (deg)	ϕ_c	20.36	18	41
Contact ratio for drive sides	m_{pd}	1.37	1.2	1.64
Contact stress (MPa)	σ_c	1090	947	1246
Pinion bending stress (MPa)	σ_{b1}	173	111	149
Gear bending stress (MPa)	σ_{b2}	237	144	242
Vibration level with frequency of the cycle of meshing (g)		7.7	2.7	10.4
Pinion tooth elastic deflection[a] (μm)	δ_1	5.7[b]	8.7	6.6
Gear tooth elastic deflection[a] (μm)	δ_2	5.1[b]	9.6	7.8

[a] Deflection of loaded tooth tip (without load sharing) calculated by FEA.
[b] Determined for virtual spur gears ($N_{1v} = 22, N_{2v} = 104$).

where $\phi_{d_{1,2}}$ is the pressure angle for drive sides in external mesh of sun and planet gears,

- for coast sides:

$$\phi_{c_{2-3}} = \arccos\left(\frac{\cos(\phi_{d_{2-3}})}{k}\right). \tag{29}$$

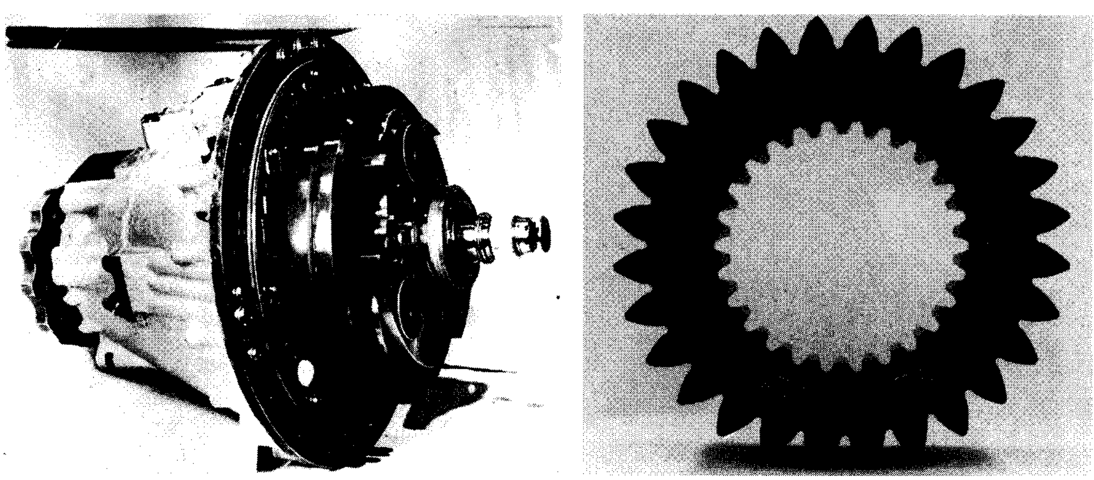

Fig. 6. The two-stage planetary gear reducer with asymmetric teeth of the turbo-prop engine TV7-117 (left) and its first stage sun gear (right).

Table 2
The turboprop engine gear reducer data

Maximum transmitted power (hp)	P	2900
Turbine shaft (RPM)		20100
Propeller (RPM)		1250
Gear reducer weight (N)		1050
Coefficient of tooth asymmetry	k	1.081
Pressure angle for drive sides in external mesh (deg)	ϕ_{d1-2}	33
Pressure angle for coast sides in external mesh (deg)	ϕ_{c1-2}	25
Pressure angle for drive sides in internal mesh (deg)	ϕ_{d2-3}	28.8
Pressure angle for coast sides in internal mesh (deg)	ϕ_{c2-3}	35.4

An application of the asymmetric teeth helped in providing the low weight to output torque ratio equal 0.063 N/(Nm) and cut down the duration and the expense of operational development.

6. Conclusions

The basic geometric theory of the gears with asymmetric teeth has been developed. This theory allows to research and design gears independently from generating rack parameters. It also provides wide variety of solutions for a particular couple of gears that are included in the area of existence. The following conclusions can be drawn from this research:

1. The asymmetric tooth geometry allows for an increase in load capacity while reducing weight and dimensions for some types of gears. It becomes possible by increasing of the pressure angle and contact ratio for drive sides.
2. General characteristics of the asymmetric gearing are optimized before the generating rack parameters computation.
3. The formulas and equations for gear and generating rack parameters are determined.
4. The formation of area of existence and versatility of asymmetric gears are shown.
5. The testing of asymmetrical gears with high pressure angle have shown significantly reduced vibration level. This can be explained by low specific sliding ratio, thick elastohydrodynamic film, and low mesh stiffness provided by the appropriate selection of coast pressure angle and fillet shape.

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